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NUMERICAL OPTIMIZATION OF BROADBAND NONLINEAR MICROWAVE CIRCUITS

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ABSTRACT

The paper describes the implementation of extensive optimization capabilities in a general-purpose harmonic-balance simulator. Two different optimizers suitable for autonomous and forced circuits, respectively, are discussed in detail, and their merits and limitations are compared. The efficient numerical optimization of nonlinear microwave circuits specified over a finite frequency band is demonstrated for the first time.

INTRODUCTION

This paper describes the implementation of extensive optimization capabilities in a previously reported general-purpose nonlinear circuit simulator {1} based on the piecewise harmonic-balance (HB) technique {2}. Two different optimizers are exploited, which are particularly suitable for autonomous and forced circuits, respectively. It is shown that the integration of both methods in a unique software package results in a very general and powerful nonlinear design tool, which can be successfully applied to the optimization of both single-frequency and broadband nonlinear microwave subsystems. The direct optimization of broadband nonlinear circuits is made possible by this package for the first time. In this way nonlinear circuits can be numerically designed to achieve given sets of performance specifications over a frequency band in much the same way as it is customary for the linear case.

The first optimization approach ("algorithm 1") was reported for the first time in {3}, and is based on the idea of treating the optimizable circuit parameters and the harmonics of the state variables (SV) used to describe the electrical regime as hierarchically equivalent problem unknowns. In other words, an objective function encompassing both the design specifications and the HB errors is simultaneously minimized with respect to both the SV harmonics and the circuit parameters {2, 3}. This leads to the simultaneous determination of a circuit topology and of a steady-state regime compatible with that topology and satisfying the design goals.

The second approach ("algorithm 2") is simply a straightforward extension of the conventional linear optimization strategy to the nonlinear-circuit case. A full nonlinear analysis of the circuit is carried out each time the objective function has to be evaluated, while the objective is minimized with respect to the circuit parameters only. This implies the nesting of two iterative procedures, which can result in severe numerical inefficiency. In practice, the use of this method is only warranted by modern techniques for the computation of the exact sensitivities of the HB errors with respect to the SV harmonics {4, 5}, and of the network functions with respect to the circuit parameters {6, 7}.

The paper reviews the fundamental algorithmic aspects of the two methods, tries to highlight the main advantages and limitations of each one, and compares their numerical performances in a typical microwave CAD application. The suitability of both in view of the implementation of an efficient, general-purpose software tool for the numerical optimization of broadband nonlinear circuits is also discussed in detail.

ALGORITHMIC ASPECTS

According to the piecewise HB technique, a generic nonlinear circuit is described as the interconnection of a linear and a nonlinear multiport. Since the latter usually consists of a set of nonlinear devices, the interconnecting ports will be referred to as the "device ports". In steady-state conditions, all time-dependent quantities have a same spectrum **S** consisting of a finite set of discrete lines. The steady states are defined by the solutions of the nonlinear algebraic system

$$\mathbf{E}(\mathbf{X}, \mathbf{P}) = \mathbf{0} \qquad (1)$$

where **E** is the set of the real and imaginary parts of all harmonic-balance errors (at all frequencies of **S** and all device ports), and **X** is the set of the real and imaginary parts of all state-variables harmonics.

For optimization purposes, a set **P** of designable circuit parameters is available in the linear subnetwork (and is explicitly shown in (1) for convenience). The vector **P** must be found in such a way that the solution of (1) satisfies a set of design goals any of which can always be expressed in the form

$$F^{(i)}_{min} \le F^{(i)}(\mathbf{X}, \mathbf{P}) \qquad (2)$$

If the HB errors are formulated in terms of current harmonics at the device ports (e.g., {8}), then both the entries of **E** and the network functions $F^{(i)}$ depend on **X** through the voltage and current harmonics at the device ports, and on **P** through the admittance matrix of the linear subnetwork, in both cases via explicitly known algebraic relationships.

In order to implement algorithm 1, each design goal of the form (2) is associated with the error function

$$E^{(i)}(\mathbf{X}, \mathbf{P}) = \begin{cases} w^{(i)} \cdot \left[F^{(i)}_{min} - F^{(i)}(\mathbf{X}, \mathbf{P}) \right] & \text{if } F^{(i)} < F^{(i)}_{min} \\ 0 & \text{if } F^{(i)}_{min} \le F^{(i)} \end{cases} \qquad (3)$$

where $w^{(i)}$ is a suitable positive weight and the index i spans all the assigned specifications. A one-sided least-p^{th} objective function {9} is then defined as

$$F_{OB}(\mathbf{X}, \mathbf{P}) = \left\{ \| \mathbf{E}(\mathbf{X}, \mathbf{P}) \|^{p} + \sum_{i} \left[E^{(i)}(\mathbf{X}, \mathbf{P}) \right]^{p} \right\}^{1/p} \qquad (4)$$

where $\|\cdot\|$ denotes the Euclidean norm and p > 0. To solve the problem, the objective is simultaneously minimized with respect to **X**, **P** until a minimum close enough to zero is reached. Note that any optimizer always proceeds by discrete steps: thus at some point during the iteration a vector **P** will be reached lying inside the region where the design goals are met. From there on, the optimizer will only work to obtain the harmonic balance, without having to compromise between the two constraints on the right-hand side of (4).

Although this algorithm was found to work satisfactorily with numerical derivatives evaluated by perturbations {8}, the implementation of exact derivatives results in a considerably improved numerical efficiency, and is thus worthwhile. The computation of the exact derivatives of the objective with respect to the SV harmonics is immediate, since general exact expressions for the derivatives of the voltage and current harmonics at the device ports wrt. the SV harmonics are available in the literature (e.g., {5}). The evaluation of the exact derivatives wrt. the circuit parameters may be carried out by conventional adjoint-network calculations {10}.

From a mathematical viewpoint, algorithm 2 can be viewed as the search for a set **P** of circuit variables for which the design goals are satisfied in the best possible way, subject to the constraint that the state lies on the manifold $\mathbf{M} \equiv [\mathbf{X} = \mathbf{X}(\mathbf{P})]$ implicitly defined by (1). In this case the HB error is guaranteed to be zero at each function evaluation, so that a generalized least-p^{th} objective {9} can be defined. Each design goal of the form (2) is now associated with the error function

$$E^{(i)}(\mathbf{X}(\mathbf{P}), \mathbf{P}) = w^{(i)} \cdot \left[F^{(i)}_{min} - F^{(i)}(\mathbf{X}(\mathbf{P}), \mathbf{P}) \right] \qquad (5)$$

Then, if E_{max} is the maximum error (in the algebraic sense), the objective function takes the form {9}

$$F_{OB}(\mathbf{P}) = \begin{cases} \left\{ \sum_i^{+} \left[E^{(i)}(\mathbf{X}(\mathbf{P}), \mathbf{P}) \right]^p \right\}^{1/p} & \text{if } E_{max} \geq 0 \\ - \left\{ \sum_i \left[-E^{(i)}(\mathbf{X}(\mathbf{P}), \mathbf{P}) \right]^{-p} \right\}^{-1/p} & \text{if } E_{max} < 0 \end{cases} \qquad (6)$$

where the superscript $^{+}$ indicates that the summation is extended to positive errors only. The objective is minimized with respect to **P**, and (1) is solved with respect to **X** prior to each function evaluation to ensure the condition $\mathbf{X} \in \mathbf{M}$.

The computation of the exact derivatives of the objective with respect to the circuit parameters is described in detail in the Appendix.

PERFORMANCE COMPARISON

Both the optimization algorithms described in the previous section were implemented in a previously reported general-purpose HB simulator {5} making use of a quasi-Newton iteration {11} to minimize the objective function. In this section we discuss the observed numerical performance.

For general nonlinear circuit optimization, algorithm 2 is usually superior to algorithm 1 in several respects. Specifically, this happens when the set of circuit configurations spanned by algorithm 2 during the iteration is entirely contained in the region of the parameter space where the Newton iteration is convergent and the solution of (1) is unique. In such cases, algorithm 2 is faster than algorithm 1 by a factor typically ranging from 4 to 8. On the other hand, if during the optimization the Newton iteration fails, the efficiency of this algorithm may be considerably impaired. It is thus clear that, for the sake of a powerful and well-conditioned optimization, any possible effort should be made in order to improve the convergence properties of the Newton iteration. In particular, this shows the interest of modern modeling techniques (e.g., {12}) allowing the dynamic range of Newton analysis to be expanded by several orders of magnitude. The essential weakness of algorithm 1 is that it tries to reach the harmonic balance by optimization, which is well known to be an inefficient approach {4}. Furthermore, algorithm 1 must find an absolute minimum (equal to zero) of its objective function, while for algorithm 2 any parameter set **P** for which the objective is negative is acceptable. Finally, algorithm 2 is better from the viewpoint of user interaction, since the iteration evolves through a sequence of physically significant circuit states, allowing the network functions to be monitored at each step. This is not true for algorithm 1, since here the harmonic balance is only reached at the end of the iteration. As an example, fig. 1 shows the results obtained from the optimization of a FET frequency doubler having the topology schematically shown in fig. 2. Curve a in fig. 1 shows the starting-point performance of the doubler, while curves b and c give the final results of single-frequency optimizations carried out with algorithm 1 and algorithm 2, respectively (4 harmonics and 4 circuit variables). Curves b and c are qualitatively comparable; however, the optimization required 190 sec with algorithm 1, and only 48 sec with algorithm 2 on a VAX 8800. Note that the analytic computation of the derivatives is essential for the numerical efficiency of both algorithms. For comparison, the optimization of the same kind of topology by algorithm 2 with an all-numerical approach (i.e., with all sensitivities evaluated by perturbations), would take about 7200 sec on the same machine {13}.

The situation is different for some very important classes of nonlinear circuits, such as oscillators (and, to some extent, frequency dividers). First of all, oscillator analysis by the Newton-iteration-based HB technique is not straightforward, and only very recently has a general solution to this problem become available {14}. Even so, the application of algorithm 2 to this kind of circuits is somewhat more problematic for the following reasons. For an oscillator the system (1) always has (i.e., for any **P**) a static solution with all but the DC components of the state vector **X** equal to zero, while solutions corresponding to oscillatory states only exist in some region of the parameter space, say **O**, which is *a priori* unknown. If the starting point $\mathbf{P}_0$ does not belong to **O** nor to its boundary, the initial state is static, and so are all the states belonging to a neighborhood of $\mathbf{P}_0$. The gradient of the objective then vanishes at the initial point, and the iteration is unable to start. Even if $\mathbf{P}_0 \in \mathbf{O}$, one has to choose an initial value of **X** for the Newton iteration which guarantees convergence to the oscillatory state, otherwise the result will be the same. Finally, in critical cases (e.g., VCOs operating near the band edge {14}), **P** may fall outside **O** during the optimization, or the Newton iteration may converge to the static solution, which may lead to severe loss of computational efficiency. All these difficulties are effectively overcome by algorithm 1 because of the circumstance that the iterations need not be solutions of (1). This implies that algorithm 1 can provide a smooth specification-driven transition from any initial state (including zero) to an oscillatory state, through a sequence of physically meaningless, but nevertheless acceptable iterations. This algorithm has been for many years the only available tool for the direct numerical optimization of microwave oscillators by the full HB approach, and has been successfully applied to many design cases, including bipolar-transistor and FET free-running oscillators {15, 16}, DROs {17} and VCOs {14}. In our present package, algorithm 2 has been integrated with algorithm 1, in order to retain the peculiar advantages of both. The optimization is started by algorithm 1, which allows a completely arbitrary initial point to be selected for both the circuit parameters and the SV harmonics. Then, after a suitable number of quasi-Newton iterations, an automatic switchover to algorithm 2 takes place, in order to exploit the superior convergence properties of the Newton iteration in the vicinity of the solution. The resulting procedure is very robust, and typically 2-3 times faster than algorithm 1 alone.

BROADBAND OPTIMIZATION

The optimization of linear circuits operating over a finite band of frequencies is available as a matter of course in all linear CAD programs. On the contrary, the numerical optimization of broadband nonlinear circuits had never been reported in the technical literature at the time of this writing. In a sense, a nonlinear circuit is always a broadband circuit because the spectrum of the steady-state waveforms always includes several discrete lines and thus covers a finite bandwidth. For our present purposes, however, we shall define as "broadband" a circuit whose performance is simultaneously specified for a number (say R) of independent steady-state regimes having different spectra $S_1, S_2, ..., S_R$. Such spectra are usually (but not necessarily) obtained from one another by changing the frequencies of one or more of the exciting sinusoidal signals. In turn, an independent state vector $\mathbf{X}_i$ is associated with each S_i, so that the set of problem unknowns includes $\mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_R$, and **P**.

Algorithm 1 can be formally extended to cover this case, simply by assuming that the index i in (3), (4) spans all the specifications at all the spectra of interest. However, this usually turns out to be impractical because of the very large number of simultaneous unknowns, and because of the need to simultaneously find the harmonic balance by optimization at a number of different spectra.

On the other hand, algorithm 2 can handle the broadband case most easily and efficiently. In this case, the state vectors $\mathbf{X}_i$ are effectively decoupled, in the sense that a separate Newton iteration with respect to each $\mathbf{X}_i$ is carried out prior to every function evaluation. As in the previous case, the objective is still defined by (5), (6) with the index i spanning all specifications and all spectra of interest. Note that this is once again a plain extension of well-known linear optimization concepts to the domain of nonlinear circuits.

Broadband optimization was implemented in our HB simulator making use of algorithm 2 with excellent results. As an example, the outcome of the broadband optimization over a 25% band of the same multiplier previously considered is shown as curve d in fig. 1. This optimization was carried out with 7 fundamental frequencies, 4 harmonics per fundamental, and 11 circuit variables, starting from the results of the narrow-band design. The optimized spectral purity of the output signal and input return loss were higher than 18 dB and 10 dB, respectively, across the band of interest. The required CPU time was about 850 sec on a VAX 8800. The numerical efficiency is thus sufficient to warrant a systematic use of this technique at the workstation level.

CONCLUSION

In conclusion, the integration of two different, and to a large extent complementary, optimization approaches into a unique harmonic-balance simulator has ensured extensive optimization capabilities of broad classes of nonlinear microwave circuits, both single-frequency and broadband, particularly in view of the ability to choose the optimization strategy that best fits any specific design problem. A major achievement of this paper is that the numerical optimization of nonlinear circuits operating over a frequency band has been demonstrated here for the first time. As shown by the example reported above, the procedure is very efficient, and thus opens the way to the straightforward design of some modern MMIC devices such as multioctave distributed mixers and power amplifiers. This effectively fulfills the gap between available linear- and nonlinear-circuit optimization capabilities.

APPENDIX

In this appendix we briefly outline the approach to the exact computation of the circuit sensitivities to be used inside algorithm 2. By means of (5) and (6), the derivatives of the objective with respect to a generic circuit parameter P are directly related to the derivatives of a generic network function $F^{(i)}$ wrt. the same quantity. Any such derivative will be denoted by the symbol D when it is taken on the manifold **M**. We have

$$\frac{DF^{(i)}}{DP} = \left(\frac{\partial F^{(i)}}{\partial \mathbf{X}}\right)^T\Bigg|_{P=const} \bullet \frac{D\mathbf{X}}{DP} + \frac{\partial F^{(i)}}{\partial P}\Bigg|_{\mathbf{X}=const} \qquad (A1)$$

The vector $\left(\frac{\partial F^{(i)}}{\partial \mathbf{X}}\right)\Big|_{P=const}$ can be computed by the composite-derivative rule. The derivatives of $F^{(i)}$ wrt. the voltage and current harmonics at the device ports are obtained explicitly from the linear subnetwork analysis, and in turn the exact derivatives of the voltage and current harmonics wrt. the state-variables harmonics are evaluated by the general formulae reported in {5}. Similarly, the scalar $\frac{\partial F^{(i)}}{\partial P}\Big|_{\mathbf{X}=const}$ is computed by first finding explicitly the derivatives of $F^{(i)}$ wrt. the linear-subnetwork admittance parameters, and then deriving the latter wrt. P by standard adjoint-network calculations {10}. Finally, by differentiating (1) we get

$$\frac{D\mathbf{X}}{DP} = -\mathbf{J}^{-1} \bullet \frac{\partial \mathbf{E}}{\partial P}\Bigg|_{\mathbf{X}=const} \qquad (A2)$$

where **J** is the Jacobian matrix of **E** with respect to **X**. Note that a factorization of this Jacobian is automatically known from the Newton iteration performed to analyze the nonlinear circuit for the current set of parameters **P**, and does not require any additional computation. If the HB errors are formulated in terms of current harmonics at the device ports (e.g., {8}), **E** is a linear function of the admittance matrix **Y** (for constant **X**), so that the vector $\frac{\partial \mathbf{E}}{\partial P}\Big|_{\mathbf{X}=const}$ is immediately expressed by means of $\frac{\partial \mathbf{Y}}{\partial P}$.

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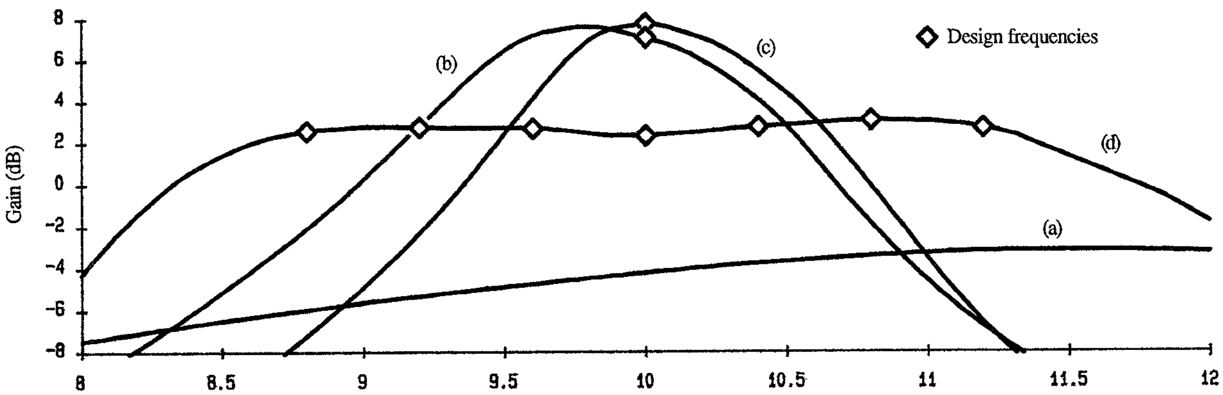

Fig. 1 - Optimization of the FET frequency doubler shown in Fig. 2 : (a) Starting point. (b) Single-frequency design by "algorithm 1". (c) Single-frequency design by "algorithm 2". (d) Broadband design.

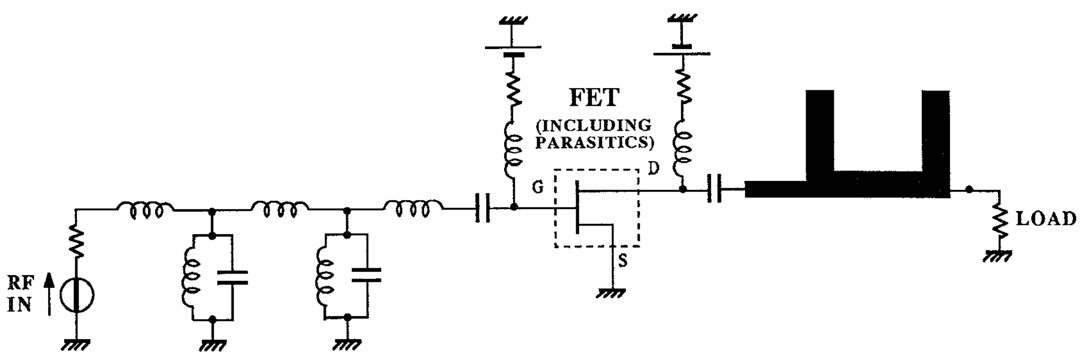

Fig. 2 - Schematic topology of a broadband FET frequency doubler